MSF Mid 1 QnA

1) What is the fermat number? find the factor of 809009 by using fermat method of factorization.

Fermat Numbers:

A number F_n is of the form

$$F_n = 2^{2^n} + 1 \quad ; \quad n \geq 0$$

is called a Fermat number.

Fermat Prime:

A Fermet number which is also a prime number is called Fermat Prime.

Examples:-

$$F_0 = 2^{2^0} + 1 = 3$$

$$F_1 = 2^{2^1} + 1 = 2^2 + 1 = 5$$

$$F_2 = 2^{2^2} + 1 = 2^4 + 1 = 17$$

$$F_3 = 2^{2^3} + 1 = 2^8 + 1 = 257$$

$$F_4 = 2^{2^4} + 1 = 2^{16} + 1 = 65537$$

Note: $F_5 = 2^{2^5} + 1 = 2^{32} + 1 = 4294967297$

is a composit number.

Factorise 809009 using format method of Factorization.

Given $n = 809009.$

find $\sqrt{n} = 899.45$

i) Let $t = \sqrt{n}+1$

$= 899+1$

$t = 900$

$\therefore \Rightarrow s^2 = t^2 - n$

$s = \sqrt{991} = 31.48$

s is not a perfect square.

ii) Let $t = \sqrt{n}+2$

$\Rightarrow t = 901$

$s^2 = t^2 - n$

$s = 52.83$

iii) Let $t = \sqrt{n}+3$

$= 899+3$

$= 902.$

$\Rightarrow s^2 = t^2 - n$

$s = 2\sqrt{698} = \cancel{52.8}.$

$= 67.78$

iv) Let $t = \sqrt{n}+4$

$= 899+4$

$= 903$

$s^2 = t^2 - n$

$s^2 = 903^2 - 3$

$s = \sqrt{903^2-3} = 80.$

$\therefore t = 904,\ s = 80$

$\therefore$ ~~903, 80 are factors of 809009.~~

$n = t^2 - s^2$

$= (t+s)(t-s)$

$= (903+80)(903-80)$

$n = (983)(823)$

$a = 983$

$b = 823$

$\therefore$ 983, 823 are the factors of 809009.

2) Solve the system of linear eq wing chinese remainder theorems x=2(mod3), x=3 (mod 5), x=2(mod 7)

Solve systems of congruences $x \equiv 2 \pmod 3$, $x \equiv 5 \pmod 5$, $x \equiv 2 \pmod 7$ using Chinese reminder Theorem. (6)

Here, $a_1 = 2$, $a_2 = 3$, $a_3 = 2$

$n_1 = 3$, $n_2 = 5$, $n_3 = 7$.

$$n = n_1 \times n_2 \times n_3$$
$$= 3 \times 5 \times 7$$
$$n = 105$$

$$N_1 = \frac{n}{n_1} = \frac{105}{3} = 35$$

$$N_2 = \frac{n}{n_2} = \frac{105}{5} = 21$$

$$N_3 = \frac{n}{n_3} = \frac{105}{7} = \cancel{150}\ 15$$

$(N_k, n_k) = 1$, $N_k x \equiv 1 \pmod{n_k}$ considaring the linear congruence:

$35x \equiv 1 \pmod 3$ —①

$21x \equiv 1 \pmod 5$ —②

$15x \equiv 1 \pmod 7$ —③

① ⇒ Let $x = 1$: $35(1) \equiv 1 \pmod 3$

$35 \equiv 1 \pmod 3$

$\frac{3}{35-1} = \frac{3}{34}$ ✗ not congruence.

Let $x = 2$: $35(2) \equiv 1 \pmod 3$

$70 \equiv 1 \pmod 3$

$\frac{3}{70-1} = \frac{3}{69}$ ✓ $\boxed{\text{Congruent}}$

$\therefore x_1 = 2.$

② ⇒ Let $x = 1$: $21(1) \equiv 1 \pmod 5$

$\frac{5}{21-1} = \frac{5}{20}$ ✓

$x_2 = 1$ $\boxed{\text{Congruent}}$

③ ⇒ Let $x = 1$:

$15(1) = 1 \pmod 7$

$\frac{7}{15-1} = 7/14$ ✓

$\boxed{\text{Congruent}}$ $x_3 = 1$

Simultanues Solution of the given System of Congruence x

$$\boxed{\bar{x} = a_1 n_1 x_1 + a_2 n_2 x_2 + a_3 n_3 x_3 .}$$

$= (2)(35)(2) + (3)(21)(1) + (2)(15)(1)$

$= 233.$

$x \equiv \bar{x} \pmod{n}$

$x \equiv 233 \pmod{105}$ $\qquad \frac{233}{\cancel{105}} = 105 \times 2 + 23$

~~Let x~~ $\Rightarrow$

$x \equiv 23 \pmod{105}$

$233 \equiv 23 \pmod{105}$

$x = 23.$

$\therefore$ $23 \equiv 2 \pmod{3}$

$23 \equiv 3 \pmod{5}$

$23 \equiv 2 \pmod{7}$

3) Fit a straight line y=ax+b for the following data

x	0	1	2	3	4
F(x)	1	1.8	3.3	4.5	6.3

The normal equations provide an alternative method for finding the coefficients (a) and (b) in the equation $(y = ax + b)$ for the least squares regression line. The normal equations are given by:

$\sum y = a \sum x + nb$

$\sum xy = a \sum x^2 + b \sum x$

values using the provided data:

$n = 5$

$\Sigma x = 0 + 1 + 2 + 3 + 4 = 10$

$\Sigma y = 1 + 1.8 + 3.3 + 4.5 + 6.3 = 16$

$\Sigma xy = (0 \cdot 1) + (1 \cdot 1.8) + (2 \cdot 3.3) + (3 \cdot 4.5) + (4 \cdot 6.3) = 44.1$

$\Sigma x^2 = (0^2) + (1^2) + (2^2) + (3^2) + (4^2) = 30$

Substitute these values into the normal equations:

$16 = a \cdot 10 + 5b$
$44.1 = a \cdot 30 + 10b$

Let's solve it step by step:

Equation 1: $16 = 10a + 5b$
Equation 2: $44.1 = 30a + 10b$

Multiply Equation 1 by 2 to make the coefficients of b match:
$32 = 20a + 10b$

Now, subtract Equation 2 from the modified Equation 1:
$32 - 44.1 = (20a + 10b) - (30a + 10b)$

Solve for a:
$-12.1 = -10a$
$a = 1.21$

Now substitute the value of a back into Equation 1 to solve for b:

$16 = 10(1.21) + 5b$
$16 = 12.1 + 5b$
$5b = 3.9$
$b = 0.78$

So, the values of a and b are 1.21 and 0.78 respectively. Therefore, the equation of the least squares regression line is
$(y = 1.21x + 0.78)$.

4) A play on tosses 3 fair coins. He wins Rs.500 of three heads, Rs 300 of two heads appear, Rs.100 if one head occurs. on the other hand He loses Rs.1500 if 3 tails occurs. find the expected gain of player.

Let X denote the gain. Then the range $x = \{-1500, 100, 300, 500\}$

The sample space $S = \{HHH, THH, HTH, THT, HTT, TTH, HHT, TTT\}$

The probability of getting all three heads: (Getting ₹500)

$P(X=3) = \frac{1}{8}$

The probability of getting 2 heads: $P(X=2) = \frac{3}{8}$ (Getting ₹300)

The probability of getting 1 head $P(X=1) = \frac{3}{8}$ (Getting ₹100)

The probability of getting no head (lossing ₹1500)

$P(X=0) = \frac{1}{8}$.

Required Descrete Probability Destribution

x	-1500	100	300	500
$P(x)$	$1/8$	$3/8$	$3/8$	$1/8$

$E(x) = \Sigma P_i \cdot x_i$

$= +1500\left(-\frac{1}{8}\right) + 100\left(3/8\right) + 300\left(\frac{3}{8}\right) + 500\left(\frac{1}{8}\right)$

$= 25$

5) find mean and variance of the uniform probability distribution given by f(x) = 1/n for x=1,2,3,...n

For a uniform probability distribution given by $f(x) = \frac{1}{n}$ for $x = 1, 2, 3, \ldots, n$, the mean (μ) and variance (σ^2) can be calculated as follows:

1. **Mean (μ):**
   The mean is the average value and is given by the formula:
   $\mu = \sum_{i=1}^{n} x_i \cdot P(x_i)$
   where x_i are the possible values of x (in this case, 1 to n) and $P(x_i)$ is the corresponding probability.

   For this uniform distribution, $P(x_i) = \frac{1}{n}$ for all x_i. Therefore, the mean is the sum of all possible values divided by n:
   $\mu = \frac{1}{n} \sum_{i=1}^{n} x_i$

   For the given distribution:
   $\mu = \frac{1}{n} \sum_{i=1}^{n} i$
   $\mu = \frac{1}{n} \sum_{i=1}^{n} i$
   $\mu = \frac{1}{n} \cdot \frac{n \cdot (n+1)}{2}$
   $\mu = \frac{n+1}{2}$
2. **Variance (σ^2):**
   The variance is a measure of the spread of the distribution and is given by:
   $\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 \cdot P(x_i)$
   where μ is the mean.

   For this uniform distribution, $P(x_i) = \frac{1}{n}$ for all x_i. Therefore, the variance is:

   $$= \frac{(n+1)(}{3}$$

   $$\text{Variance} = \sum p_i x_i^2 - \mu^2$$

   $$= \frac{1}{n}\left[1^2 + 2^2 + 3^2 + \ldots + n^2\right] - \left(\frac{n+1}{2}\right)^2$$

   $$= \frac{1}{\cancel{n}} \frac{\cancel{n}(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n+}{}$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

$$= \frac{(n+1)(n-1)}{12}$$

$$\sigma^2 = \frac{n^2-1}{12}.$$

$$\therefore \text{Mean} = \frac{n+1}{2}, \quad \text{Variance} = \frac{n^2-1}{12}$$

Therefore, the mean μ is $\frac{n+1}{2}$ and the variance σ^2 is $\frac{n^2-1}{12}$ for the given uniform probability distribution.

6) A sample of 4 items is selected at random from a box containing 12 items of which 5 are defective. Find the expected number 'E' of defective items.

34 495

Let X denote the number of defective items 175

among 4 items drawn from 12 items 210

X can take the values 0,1,2,3,4. 70

5

The number of defective items = 5

The no. of ~~doo~~ non-defective items = 7.

$P(X=0) = P(\text{no. of defective} = 0) = \frac{{}^7C_4\, {}^5C_0}{{}^{12}C_4} = \frac{35}{495}$ $\frac{7}{99}$

$$\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}}$$

${}^nC_1 = n$ ${}^nC_n = 1$

${}^nC_0 = 1$

$P(X=1) = P(\text{one defect}) = \frac{{}^7C_3\, {}^5C_1}{{}^{12}C_4} = \frac{175}{495}$ $\frac{35}{99}$

$P(X=2) = P(\text{two defect}) = \frac{{}^7C_2\, {}^5C_2}{{}^{12}C_4} = \frac{210}{495} = \frac{42}{99}$ $\frac{14}{33}$

$P(X=3) = P(\text{three defect}) = \frac{{}^7C_1\, {}^5C_3}{{}^{12}C_4} = \frac{70}{495}$ $\frac{14}{99}$

$$P(X=4) = P(\text{all defects}) = \frac{{}^7C_0 \, {}^5C_4}{{}^{12}C_4} = \frac{5}{495} = \frac{1}{99}$$

$1.\overline{666}$

$3.\overline{4X}$

$0.70\overline{70}$

Required Probability Distribution.

X	0	1	2	3	4
P(x)	$\frac{7}{99}$	$\frac{35}{99}$	$\frac{42}{99}$	$\frac{14}{99}$	$\frac{1}{99}$.

$$\text{Mean (Expectation)} = \sum_{i=0}^{4} P_i X_i$$

$$\mu = 0 + \frac{35}{99} + \frac{84}{99} + \frac{42}{99} + \frac{4}{99}$$

$$\mu = 1.\overline{6}$$

$$\text{Variance} = \sum_{i=0}^{4} P_i x_i^2 - \mu^2$$

$$= 0 + \frac{35}{99} + \frac{4 \times 42}{99} + \frac{9 \times 14}{99} + \frac{16}{99} - (1.\overline{6})^2$$

$$= 3.\overline{48} - 1.\overline{6}^2 = 0.\overline{70}$$

7) What is the fermat number? find the factor of 426749 by using formal method of factorization.

Format Numbers:

A number F_n is of the form

$$F_n = 2^{2^n} + 1 \quad ; \quad n \geq 0$$

is called a Format number.

Fermat Prime:

A Fermet number which is also a prime number is called Format Prime.

Examples:-

$$F_0 = 2^{2^0} + 1 = 3$$

$$F_1 = 2^{2^1} + 1 = 2^2 + 1 = 5$$

$$F_2 = 2^{2^2} + 1 = 2^4 + 1 = 17$$

$$F_3 = 2^{2^3} + 1 = 2^8 + 1 = 257$$

$$F_4 = 2^{2^4} + 1 = 2^{16} + 1 = 65537$$

Note: $F_5 = 2^{2^5} + 1 = 2^{32} + 1 = 4294967297$ is a composit number.

factor of 426749 by using formal method of factorization:

1. **Choose t:**

   Let's choose $t = 655$ (you can experiment with different values).

2. **Check $t^2 - N$:**

   Calculate $t^2 - N$ and check if it's a perfect square. If $t^2 - N = s^2$ for some s, then N can be factored as $(t+s)(t-s)$.

   $656^2 - 426749 = 430336 - 426749 = 3587$

   Since 3587 is not a perfect square, we need to choose a different value for t and repeat the process.

3. **Choose a Different t:**

   Let's try another value. Let's choose $t = 657$.

4. **Check $t^2 - N$:**

   $657^2 - 426749 = 431649 - 426749 = 4900$

   Now, $t^2 - N = 4900$, which is a perfect square (70^2).

5. **Factorize Using the Result:**

   Since $s^2 = t^2 - N = 4900$ and $4900 = 70^2$, we can write N as the difference of two squares:

   $N = t^2 - s^2 = (t + 70)(t - 70)$

   Therefore, the factors of 426749 are $t + 70$ and $t - 70$.

6. **Calculate Factors:**

   $a + 70 = 657 + 70 = 727$

   $a - 70 = 657 - 70 = 587$

Therefore, the factors of 426749 are 587 and 727.

8) solve the system of linear equation using chinese remainder theorem x=5 (mod 6), 1 = 4 (mod 11) x = 3 (mod17)

i) $x \equiv 5 \pmod 6$ comparing to $x \equiv a_1 \pmod{n_1}$

$x \equiv 4 \pmod{11}$ $x \equiv a_2 \pmod{n_2}$

$x \equiv 3 \pmod{17}$ $x \equiv a_3 \pmod{n_3}$

$a_1 = 5,\ a_2 = 4,\ a_3 = 3$

$n_1 = 6,\ n_2 = 11,\ n_3 = 17$

$N = n_1 \times n_2 \times n_3$

$N = 6 \times 11 \times 17$

$N = 1122$

$N_1 = \frac{N}{n_1} \Rightarrow N_1 = \frac{1122}{6} = 187$

$N_2 = \frac{N}{n_2} \Rightarrow N_2 = \frac{1122}{11} = 102$

$N_3 = \frac{N}{n_3} \Rightarrow N_3 = \frac{1122}{17} = 66$

Here, $(N_k, n_k) = 1$ i.e, $N_k \cancel{x} \equiv 1 \pmod{n_k}$

$\Rightarrow$

$187 x_1 \equiv 1 \pmod 6$ — (1)

$102 x_2 \equiv 1 \pmod{11}$ — (2)

$66 x_3 \equiv 1 \pmod{17}$ — (3)

For Eq (1)

i) If $x_1 = 1$: $187(1) \equiv 1 \pmod 6$

$\Rightarrow 187 \pmod 6 = 1$

$1 = 1$

It is congruent

For Eq (2)

i) If $x_2 = 1$: $102(1) \equiv 1 \pmod{11}$

$\Rightarrow 102 \pmod{11} = 1$

$3 \neq 1$

~~not~~ not congruent

ii)

If $x_2 = 2 \Rightarrow 102(2) \equiv 1 \pmod{11}$

If $x_2 = 2$: $102(2) \equiv 1 \pmod{11}$

$x_1 = 1$

$204 \pmod{11} = 1$

$6 \neq 1$

not congruent

iii) If $x_2 = 3$: $102(3) \equiv 1 \pmod{11}$

$306 \pmod{11} = 1$

$9 \neq 1$

It is not congruent

iv) If $x_2 = 4$: $102(4) \equiv 1 \pmod{11}$

$408 \pmod{11} = 1$

$1 = 1$

It is congruent

$x_2 = 4.$

For Eq (3)

i) If $x_3 = 1$: $66(1) \equiv 1 \pmod{17}$

$66 \pmod{17} = 1$

$15 \neq 1$

not congruent

ii) If $x_3 = 2$: $66(2) \equiv 1 \pmod{17}$

$132 \pmod{17} = 1$

$13 \neq 1$

not congruent

iii) If $x_3 = 7$: $66(7) \equiv 1 \pmod{17}$

$462 \pmod{17} = 1$

$3 \neq 1$

not congruent

iv) If $x_3 = 8$: $66(8) \equiv 1 \pmod{17}$

$528 \pmod{17} = 1$

$1 = 1$

It is congruent

$x_3 = 8$

Hence,

$x_1 = 1$

$x_2 = 4$

$x_3 = 8$

~~and~~ $a_1 = \cancel{5}\ 187$, $a_2 = \cancel{4}\ 102$, $a_3 = \cancel{3}\ 66$

$n_1 = 6$, $n_2 = 11$, $n_3 = 17$

according to simultaneous solution of the given system of congruence.

$$\bar{x} = a_1 N_1 x_1 + a_2 N_2 x_2 + a_3 N_3 x_3$$

$$x = \bar{x} \pmod N \quad (\text{or}) \quad \bar{x} \equiv x \pmod N$$

$a_1 = 5, \; a_2 = 4, \; a_3 = 3$

$N_1 = 187, \; N_2 = 102, \; N_3 = 66$

$x_1 = 1, \; x_2 = 4, \; x_3 = 8$

$\bar{x} = 5(187)(1) + 4(102)(4) + 3(66)(8)$

$\bar{x} = 935 + 1632 + 1584$

$\bar{x} = 4151$

$\because N = \cancel{1122}\ 1122$

$4151 \equiv x \pmod{1122}$

$x = 4151 \pmod{1122}$

$x = 785$

Therefore, The system of linear equation is

$785 \equiv 5 \pmod 6$

$785 \equiv 4 \pmod{11}$

$785 \equiv 3 \pmod{17}$

9) Out of 800 families with 5 children each how would many you expect to have a) 3 boys b)5 girls c) either 2 (or) 3 boys d) atleast one boy? Assume equal properties probabilities for boys and girls.

~~It~~ let probability of no. of boys in each family be $p = \frac{1}{2}$, of girl $q = \frac{1}{2}$.

no. of boys in family $= X$.

no. of children $\Rightarrow n = 5$.

i) Probability of having 3 boys:

$P(X = 3)$.

$$P(X=3) = {}^{5}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$$

$$= \frac{5}{16}$$

For 800 families, the ~~pro~~ probability of no. of families having 3 boys:

$$\Rightarrow \frac{5}{16} \times 800 = 250 \text{ families.}$$

ii) 5 girls

$$P(X=0) = {}^{5}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0}$$

$$= \frac{1}{32}.$$

For 800 familes, the no. of families having 5 girls

i.e, $\frac{1}{32} \times 800 = 25$

iii) 2 or 3 boys

$= P(X=2) + P(X=3)$

$= {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2} + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$

$= \frac{5}{16} + \frac{5}{16}$

$= \frac{10}{16} = \frac{5}{8}$

For 800 families, the probability of no. of families having 2 or 3 boys is $\frac{10}{16} \times 800$

iv) Atleast one boy

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - \frac{1}{32}$

$= \frac{31}{32}.$

For 800 families the probability of no. of families having atleast one boy is

$\frac{31}{32} \times 800 = 775.$

10) Write the proof mean of normal distribution.

Mean of the Normal Distribution :-

By the definition of Normal distribution,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty$$

$$\text{Mean } \mu = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x\, e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)} dx$$

Let $\frac{x-\mu}{\sigma} = z \Rightarrow x-\mu = z\sigma$

$x = z\sigma + \mu$

$dx = \sigma\, dz + 0.$

$$\Rightarrow \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z + \mu)\, e^{\frac{-1}{2} z^2} (\cancel{\sigma}\, dz)$$

$$= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z\, e^{-\frac{z^2}{2}} dz + \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2} dz$$

$$= 0 + \frac{2\mu}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{z^2}{2}} dz$$

Let $\frac{z^2}{2} = t \Rightarrow z = \sqrt{2t}$

$t^2 = 2t$

$2t\,dt = 2\,dt \quad dt = \frac{dt}{t} \qquad dz = \frac{dt}{\sqrt{2t}}$

$$= \frac{2\mu}{\sqrt{2\pi}} \int_0^\infty e^{-t} \frac{dt}{\sqrt{2t}} \qquad \left[\because \Gamma n = \int_0^\infty e^{-x} x^{n-1} dt\right]$$

$$\frac{2\mu}{\sqrt{2}\sqrt{2\pi}} \int_0^\infty e^{-t} t^{-1/2} dt$$

$n-1 = -1/2$
$n = 1 - 1/2$
$n = 1/2$

$$\frac{\mu}{\sqrt{\pi}} \int_0^\infty e^{-t} t^{1/2-1} dt$$

$$= \frac{\mu}{\sqrt{\pi}} \Gamma\tfrac{1}{2} \qquad \left[\because \Gamma\tfrac{1}{2} = \sqrt{\pi}\right]$$

$$= \frac{\mu}{\sqrt{\pi}} \sqrt{\pi}$$

Mean $= \mu$

11) Write proof variance of normal distribution

Variance of Normal Distribution:

By the definition of Normal Distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$
$$-\infty < \mu < \infty$$

Variance $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^2 e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Put $\frac{x-\mu}{\sigma} = z \Rightarrow$ $x = \sigma z + \mu$

$dx = \sigma\, dz$.

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z)^2 e^{-\frac{1}{2}(z)^2} (\cancel{\sigma}\, dz)$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-z^2/2}\, dz .$$

Put $\frac{z^2}{2} = t$

$z = \sqrt{2t}$

$dz = \frac{dt}{\sqrt{2t}}$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^{\infty} z^2 e^{-z^2/2}\, dz$$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^{\infty} (\sqrt{2t})^2 e^{-t} \frac{dt}{\sqrt{2t}}$$

$$= \frac{2 \times 2 \times \sigma^2}{\sqrt{2}\sqrt{2}\pi} \int_0^{\infty} e^{-t} \frac{t}{t^{1/2}}\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_0^{\infty} e^{-t}\, t^{3/2 - 1}\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \Gamma\left(\tfrac{3}{2}\right) \qquad \left[\because \Gamma\left(\tfrac{3}{2}\right) = \tfrac{1}{2}\Gamma\left(\tfrac{1}{2}\right) = \tfrac{1}{2}\sqrt{\pi}\right]$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{2}\sqrt{\pi} .$$

$n - 1 = 1/2$

$n = \frac{1}{2} + 1$

$n = 3/2$

$\Gamma(n) = (n-1)\Gamma(n-1)$

Variance $= \sigma^2$

calculate expectation and variance of 'X' variance of the probability distribution of the random variable 'X' is given by

x	-1	0	1	2	3
F(x)	0.3	0.1	0.1	0.3	0.2

Given the probability distribution of X:

x	-1	0	1	2	3
F(x)	0.3	0.1	0.1	0.3	0.2

1. Expectation (μ):
   $\mu = E(X) = \sum_i x_i \cdot P(X = x_i)$
   $\mu = (-1 \cdot 0.3) + (0 \cdot 0.1) + (1 \cdot 0.1) + (2 \cdot 0.3) + (3 \cdot 0.2)$
   $\mu = -0.3 + 0 + 0.1 + 0.6 + 0.6$
   $\mu = 1$
2. Variance (σ^2):
   $\sigma^2 = \sum_i x_i^2 \cdot P(X = x_i) - \mu^2$

Given that $\mu = 1$ (calculated previously), let's use this formula to find the variance:

$$\sigma^2 = \sum_i x_i^2 \cdot P(X = x_i) - \mu^2$$
$$\sigma^2 = (-1)^2 \cdot 0.3 + (0)^2 \cdot 0.1 + (1)^2 \cdot 0.1 + (2)^2 \cdot 0.3 + (3)^2 \cdot 0.2 - 1^2$$
$$\sigma^2 = 0.3 + 0 + 0.1 + 1.8 + 1.2 - 1$$
$$\sigma^2 = 2.4$$

So, the expectation (μ) is 1, and the variance (σ^2) is 2.4 for the given probability distribution.

12) A sample of u items is selected a vandom from a box containing 12 items of which 5 are defective. Find

the expected number 'E' of defective items

// Same Question 6

13) In normal distribution define define mean = median = mode

// Not given